

Graphene induced mode bifurcation at low input power

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We study analytically the plasmonic modes in the graphene-coated dielectric nanowire, based on the explicit form of nonlinear surface conductivity of graphene. The propagation constants of different plasmonic modes can be tuned by the input power at the order of a few tenths of mW. The lower and upper mode bifurcation branches are connected at the limitation value of the input power. Moreover, due to the nonlinearity of graphene, the dispersion curves of plasmonic modes at different input powers form an energy band, which is in sharp contrast with the single dispersion curve in the limit of zero input power.

Nonlinear plasmonics is a newly developed but explosive growing field which not only offers extreme light manipulation at the subwavelength scale [1], but also provides an universal method to scale down the conventional nonlinear optical devices to the chip scale [2]. Due to the strong local electromagnetic fields, nonlinear plasmonic effects can originate from the adjacent nonlinear dielectric media. In the past years, nonlinear plasmonic modes in metal-dielectric[3–6], dielectric-metal-dielectric[7–11], and metal-dielectric-metal [12, 13] planar structures have been studied extensively. Meanwhile, the existence of discrete solitons in nonlinear dielectric media embedded with periodic metallic films [14], nanowires [15], and nanorings [16] have been proposed recently.

As a newly discovered two dimensional electromagnetic material, graphene has received extensive attention in nonlinear plasmonics. Compared with ordinary dielectric media, graphene has a high nonlinear susceptibility [19–23], which is promising to release the demand of high input power in current nonlinear plasmonics. Until now, some basic phenomena based on graphene induced nonlinearities have been studied, e.g., solitons supported by monolayer graphene [24] and multilayer graphene [25–27]. However, little attention has been paid to the two dimensional graphene-based nonlinear plasmonic waveguides, although two dimensional structures are more favourable as fundamental building blocks of plasmonic waveguide arrays and plasmonic lattices [15, 16].

In this Letter, we give explicitly the tensor form of nonlinear surface conductivity of graphene in the classical frequency range. As a simplest structure of the two dimensional waveguides, the plasmonic modes with different orders are presented analytically in the graphene-coated dielectric nanowire, where the nonlinearity of graphene is considered. Meanwhile, the dependencies between the propagation constants and the input power are discussed. Moreover, the energy band which is formed by dispersion curves at different input powers, is studied by taking the fundamental mode as an example.

Due to the two dimensional nature of graphene, its third order surface conductivity is a fourth-order tensor with

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16 elements. Previous studies mainly focus on the element $\sigma_{xxxx}^{(3)}$ [19, 20, 28], whereas other elements are needed when the incident electric field has two components along the graphene surface. For this reason, we assume that the doped graphene monolayer is placed on the xy plane and a time-dependent electric field of the form $\mathbf{E}(t) = [E_x \exp(-i\omega t) + c.c.] \hat{x} + [E_y \exp(-i\omega t) + c.c.] \hat{y}$ is applied, where ω is the angular frequency of the electromagnetic field. Basically the optical response of graphene is contributed by both the intraband and interband electronic transitions [22, 23]. However, in the classical frequency range $\hbar\omega \leq \mu_c$, namely the photon energy is smaller than the chemical potential, the optical response of graphene is dominated by the intraband transitions. Under the relaxation time approximation and neglecting the interband processes, the transport properties of electrons in graphene are governed by the following Boltzmann equation

$$\frac{\partial f(\mathbf{k}, t)}{\partial t} - \frac{e}{\hbar} \mathbf{E}(t) \cdot \frac{\partial f(\mathbf{k}, t)}{\partial \mathbf{k}} = -\frac{f(\mathbf{k}, t) - f_0(\mathbf{k})}{\tau}, \quad (1)$$

where $\mathbf{k} = (k_x, k_y)$ is the wave vector, $f(\mathbf{k}, t)$ is the nonequilibrium distribution function, $f_0(\mathbf{k}) = 1/[1 + e^{(\epsilon(\mathbf{k}) - \mu_c)/k_B T}]$ is the equilibrium Fermi-Dirac distribution function, $\epsilon(\mathbf{k}) = v_F \hbar \sqrt{k_x^2 + k_y^2}$ is the Dirac cone spectrum of charge carriers in graphene, $v_F = c/300$ is the Fermi velocity, c is the velocity of light in free space, τ is the relaxation time, $-e$ is the charge of an electron, \hbar is the reduced Planck's constant, k_B is the Boltzmann's constant, and T is the temperature.

The exact solution of Eq. (1) at $\omega\tau \gg 1$ is [27, 28]

$$f(\mathbf{k}, t) = \frac{e^{-t/\tau}}{\tau} \int_{-\infty}^t dt' e^{t'/\tau} f_0[\mathbf{k} + \kappa(t, t')], \quad (2)$$

where $\kappa(t, t') = \frac{e}{\hbar} \int_{t'}^t \mathbf{E}(t'') dt''$. The surface current along the graphene surface can be expressed as

$$\mathbf{j}(t) = -4 \frac{e}{(2\pi)^2 \hbar} \int d\mathbf{k} f(\mathbf{k}, t) \frac{\partial \epsilon(\mathbf{k})}{\partial \mathbf{k}}, \quad (3)$$

where the factor 4 is due to spin degeneracy and valley degeneracy [27]. In the low temperature limit $T \rightarrow 0$, the nonequilibrium distribution function can be replaced by the Heaviside step function. Thus the surface current reduces to

$$j_i(t) = -\frac{ev_F}{\pi^2 \tau} e^{-t/\tau} \int_{-\infty}^t dt' e^{t'/\tau} I_i(t, t'), \quad (4)$$

where

$$I_i(t, t') = \int d\mathbf{k} \frac{k_i}{\sqrt{k_x^2 + k_y^2}} H[\mu_c - \epsilon(\mathbf{k} + \kappa(t, t'))], \quad (5)$$

and $i = x, y$. Eq. (5) can be calculated by expanding the integrand with respect to $\kappa_x(t, t')$ and $\kappa_y(t, t')$ (up to the third order) and integrating over the Fermi surface. Calculation shows that

$$I_i(t, t') = -k_F \pi \kappa_i(t, t') + \frac{\pi}{8k_F} \kappa_i(t, t') \kappa^2(t, t'), \quad (6)$$

where $k_F = \mu_c/v_F \hbar$ is the Fermi wave vector. Without loss of generality, we consider the surface current in x direction. According to Eqs. (4) and (6), and considering the equivalence between x and y coordinates, we obtain

$$j_x = j_x(\omega) \exp(-i\omega t) + j_x(3\omega) \exp(-i3\omega t) + c.c., \quad (7)$$

where the first term corresponds to the surface current with time dependence $\exp(-i\omega t)$ and the second term corresponds to the third harmonics. In what follows, we neglect the terms of third harmonics since the phase matching condition is required[31], and obtain

$$\begin{aligned} j_x(\omega) = & \sigma_{xx}^{(1)} E_x + [\sigma_{xxyy}^{(3,\omega)} + \sigma_{xyyx}^{(3,\omega)}] E_x |E_y|^2 + \sigma_{xyxy}^{(3,\omega)} E_x^* E_y^2 \\ & + \sigma_{xxxx}^{(3,\omega)} |E_x|^2 E_x. \end{aligned} \quad (8)$$

Thus $\sigma_{xx}^{(1)} = \sigma_{yy}^{(1)} = 4\sigma_0 \mu_c / \pi \hbar (1/\tau - i\omega)$ is the linear surface conductivity, $\sigma_{xxyy}^{(3,\omega)} = \sigma_{xyxy}^{(3,\omega)} = \sigma_{xyyx}^{(3,\omega)} = \sigma_{yyxx}^{(3,\omega)} = \sigma_{yxxy}^{(3,\omega)} = \sigma_{yxyx}^{(3,\omega)} = \sigma_{xxxx}^{(3,\omega)} / 3 = \sigma_{yyyy}^{(3,\omega)} / 3 = -3\sigma_0 e^2 v_F^2 / \pi \mu_c \hbar (1/\tau^2 + \omega^2) (1/\tau - i2\omega)$ are the nonzero elements of nonlinear surface conductivity, and $\sigma_0 = e^2 / 4\hbar$ is the conductivity quantum.

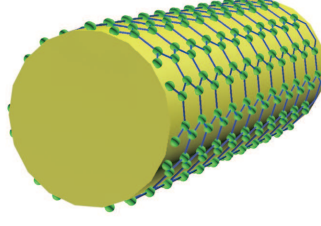


FIG. 1: Structure of the graphene-coated dielectric nanowire.

Under the limit of $\omega\tau \gg 1$, namely the relaxation time is large compared with the oscillation period of the incident electromagnetic field, the surface conductivity of graphene is

$$\sigma = \sigma^{(1)} + \sigma^{(3)} |\mathbf{E}_{\parallel}|^2, \quad (9)$$

where

$$\sigma^{(1)} = i \frac{e^2 \mu_c}{\pi \hbar^2 \omega} \quad (10)$$

is the linear part of surface conductivity,

$$\sigma^{(3)} = -i \frac{9e^4 v_F^2}{8\pi \mu_c \hbar^2 \omega^3} \quad (11)$$

is the nonlinear part of surface conductivity, and \mathbf{E}_{\parallel} is the electric field that is parallel to the graphene surface. Note in deriving Eqs. (9)-(11), we have used the condition $E_y^* E_x = E_x^* E_y$, which requires that the electric field \mathbf{E}_{\parallel} is linearly polarized. If the incident wave is circularly polarized or elliptically polarized, the tensor form of nonlinear surface conductivity should be used.

Besides, Eqs. (9)-(11) are valid under the approximation conditions of $\hbar\omega \leq \mu_c$ and $\omega\tau \gg 1$. The carrier relaxation time τ is determined by the carrier mobility μ as $\tau = \mu \mu_c / e v_F^2$, where the carrier mobility μ of graphene film ranges from $1000 \text{ cm}^2/(\text{V} \cdot \text{s})$ in chemical vapor deposition (CVD)-grown graphene [29] to $230\,000 \text{ cm}^2/(\text{V} \cdot \text{s})$ in suspended exfoliated graphene [30]. When using a moderate mobility of $10\,000 \text{ cm}^2/(\text{V} \cdot \text{s})$ and with $\mu_c = 0.3 \text{ eV}$, the frequency of the electric field satisfies $0.5 \text{ THz} \ll \omega/2\pi \leq 45.3 \text{ THz}$.

Considering the practical applications where two dimensional structures are more favourable, in the following we calculate the plasmonic modes of graphene-coated dielectric nanowire, where the structure is shown in Fig. 1. For the modes propagating in the axial direction of the waveguide, the field can be expressed as

$$\tilde{\mathbf{A}}(r, \theta, z, t) = \mathbf{A}(r, \theta) \exp[i(\beta z - \omega t)] + c.c., \quad (12)$$

where $\tilde{\mathbf{A}}$ denotes the electric field $\tilde{\mathbf{E}}$ or the magnetic field $\tilde{\mathbf{H}}$, β is the propagation constant, and z is the propagation direction. From Eq. (12), the z component of the field satisfies the following equation

$$\frac{\partial^2 A_z}{\partial r^2} + \frac{1}{r} \frac{\partial A_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \theta^2} - (\beta^2 - k_0^2 \varepsilon) A_z = 0, \quad (13)$$

where $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$, and ε is the relative permittivity of the material ($\varepsilon = \varepsilon_1$ for the inside dielectric nanowire and $\varepsilon = \varepsilon_2 = 1$ for the outside air). Using the method of separating variables, the solutions for the m -th order plasmonic mode are

$$E_z(r, \theta) = i \frac{\sqrt{P_0 \eta_0}}{a} A_m I_m \left(u \frac{r}{a} \right) e^{im\theta}, \quad (14)$$

$$H_z(r, \theta) = \frac{\sqrt{P_0 / \eta_0}}{a} B_m I_m \left(u \frac{r}{a} \right) e^{im\theta}, \quad (15)$$

for $r \leq a$, and

$$E_z(r, \theta) = i \frac{\sqrt{P_0 \eta_0}}{a} C_m K_m \left(w \frac{r}{a} \right) e^{im\theta}, \quad (16)$$

$$H_z(r, \theta) = \frac{\sqrt{P_0 / \eta_0}}{a} D_m K_m \left(w \frac{r}{a} \right) e^{im\theta}, \quad (17)$$

for $r > a$, where a is the radius of the dielectric nanowire, $u = a\sqrt{\beta^2 - k_0^2\varepsilon_1}$, $w = a\sqrt{\beta^2 - k_0^2\varepsilon_2}$, A_m , B_m , C_m and D_m are the dimensionless undetermined constants, $P_0 = \frac{1}{4} \iint_S (\mathbf{E} \times \mathbf{H}^* + \mathbf{E}^* \times \mathbf{H}) \cdot \hat{z} dS$ is the input power, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the impedance of free space, and I_m and K_m are the m -th order modified Bessel function of the first kind and the second kind, respectively. Utilizing the above results, the other components of the electric field and magnetic field can be obtained from Maxwell equations [32]. Since the electric field that is parallel to the graphene surface is linearly polarized which satisfies $E_y^* E_x = E_x^* E_y$, we can use Eqs. (9)-(11) to characterize the surface conductivity of graphene directly. Besides, since there are two components along the graphene surface, the contributions from both two components should be considered, which is different to the case of planar structures [24, 26, 27]. According to the continuity conditions at $r = a$ and at arbitrary θ , we can get the following equations

$$A_m I_m(u) = C_m K_m(w), \quad (18)$$

$$B_m I_m(u) - D_m K_m(w) = i \frac{\sigma \eta_0 a}{u^2} [\beta m A_m I_m(u) + k_0 u B_m I_m'(u)], \quad (19)$$

$$\frac{1}{u^2} [\beta m A_m I_m(u) + k_0 u B_m I_m'(u)] = \frac{1}{w^2} [\beta m C_m K_m(w) + k_0 w D_m K_m'(w)], \quad (20)$$

$$\begin{aligned} & \frac{1}{u^2} [k_0 \varepsilon_1 u A_m I_m'(u) + \beta m B_m I_m(u)] \\ & - \frac{1}{w^2} [k_0 \varepsilon_2 w C_m K_m'(w) + \beta m D_m K_m(w)] \\ & = -i \frac{\sigma \eta_0}{a} A_m I_m(u), \end{aligned} \quad (21)$$

where

$$\sigma = \sigma^{(1)} + \sigma^{(3)} |\mathbf{E}_\parallel|^2 = \sigma^{(1)} + \sigma^{(3)} (|E_z(a)|^2 + |E_\theta(a)|^2), \quad (22)$$

$$E_z(a) = i \frac{\sqrt{P_0 \eta_0}}{a} A_m I_m(u), \quad (23)$$

$$E_\theta(a) = i \frac{\sqrt{P_0 \eta_0}}{u^2} [\beta m A_m I_m(u) + k_0 u B_m I_m'(u)], \quad (24)$$

and $m = 0, 1, \dots$. Meanwhile, since the input power is P_0 , we obtain the normalization condition as follows

$$\begin{aligned} & \frac{\pi \beta k_0}{u^4} (\varepsilon_1 A_m^2 + B_m^2) \int_0^a \left(\frac{m^2 a^2}{r^2} I_m^2 + u^2 I_m'^2 \right) r dr \\ & + \frac{2\pi a}{u^3} A_m B_m (\varepsilon_1 k_0^2 + \beta^2) \int_0^a I_m I_m' dr \\ & + \frac{\pi \beta k_0}{w^4} (\varepsilon_2 C_m^2 + D_m^2) \int_a^\infty \left(\frac{m^2 a^2}{r^2} K_m^2 + u^2 K_m'^2 \right) r dr \\ & + \frac{2\pi a}{w^3} C_m D_m (\varepsilon_2 k_0^2 + \beta^2) \int_a^\infty K_m K_m' dr = 1. \end{aligned} \quad (25)$$

Thus, from the continuity conditions (18)-(24) and the normalization condition (25), we can solve the five unknown parameters A_m , B_m , C_m , D_m and β numerically. Specially, when the nonlinear surface conductivity of graphene is neglected, namely $\sigma^{(3)} = 0$, our nonlinear plasmonic waveguide reduces to the common linear plasmonic waveguide, and Eqs. (18)-(25) reduces to the dispersion relation of linear plasmonic modes, which have been discussed in Refs. [32, 33].

In what follows, we let $f = \omega/2\pi = 30$ THz, $\mu_c = 0.3$ eV, $a = 100$ nm, and $\varepsilon_1 = 3$. Since the relaxation time of graphene ranges from 0.01 ps to 1 ps [23], our parameters fulfill the approximation conditions of $\hbar\omega \leq \mu_c$ and $\omega\tau \gg 1$. As shown in Fig. 2, graphene-coated dielectric nanowire supports four orders of plasmonic modes, which correspond to $m = 0, 1, 2$, and 3, respectively. Note when the input power tends to zero, the propagation constants reduce to that of the linear plasmonic modes [32], where the nonlinear surface conductivity of graphene is neglected. In other words, the linear plasmonic modes in Refs. [32] are approximations of our nonlinear plasmonic modes in the limit of zero input power.

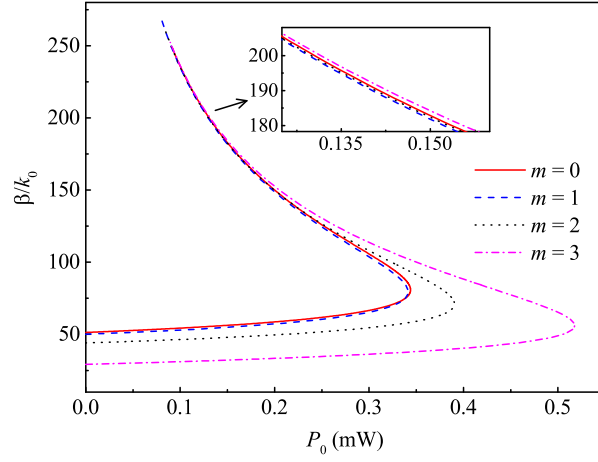


FIG. 2: Mode bifurcation curves with different orders. The inset is the enlarged figure. The parameters are $f = 30$ THz, $\mu_c = 0.3$ eV, $a = 100$ nm, and $\varepsilon_1 = 3$.

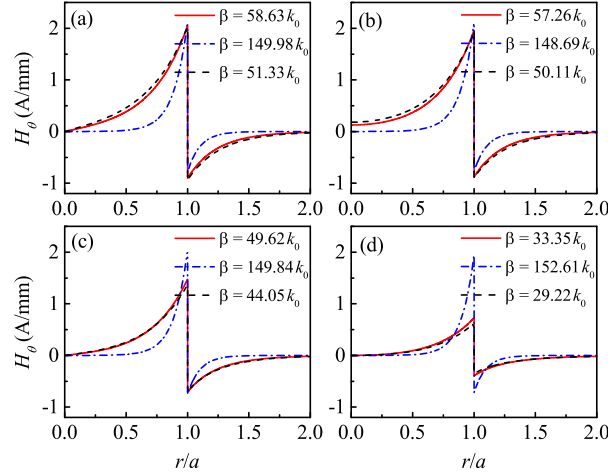


FIG. 3: The distribution of plasmonic modes in the radial direction at $\theta = 0$ for (a) $m = 0$, (b) $m = 1$, (c) $m = 2$, and (d) $m = 3$, respectively. For comparison, the linear plasmonic modes are also plotted (black dashed curves). The parameters are $f = 30$ THz, $\mu_c = 0.3$ eV, $a = 100$ nm, $\varepsilon_1 = 3$, and $P_0 = 0.2$ mW.

Starting from the zero power point, as the input power increases along the lower bifurcation branch, the field intensity at the graphene surface also increases. According to Eqs. (9)-(11), the surface conductivity of graphene decreases accordingly. Thus, the propagation constant of plasmonic mode increases with the enhancement of the field confinement, as shown in Fig. 3. For comparison, we also plot the corresponding linear plasmonic modes. Clearly, the propagation constants of these linear modes are equal to that of the nonlinear plasmonic modes in the limit of zero input power.

In the lower bifurcation branch, the propagation constant increases monotonically with the input power, when the input power is below a limitation value. This limitation value is the maximum value of the allowed input power, where the m th mode only exists when the input power is below its corresponding limitation value. However, the field confinement can be enhanced further if the input power decreases from the limitation value along the upper bifurcation branch. Although the input power is decreased, the field intensity at the graphene surface is increased which insures the continuous growing of the propagation constant, as shown in Figs. 2 and 3. Note although there are intersection points when $P_0 = 0.339$ mW for the curves with $m = 0$ and $m = 1$, and $P_0 = 0.213$ mW for the curves with $m = 0$ and $m = 2$, degenerate states do not exist since these modes belong to different orders. Moreover, due to the high nonlinear surface conductivity of graphene, the propagation constants for different orders can be tuned by the input power at the order of a few tenths of mW, which cannot be realized by conventional nonlinear dielectric media [23]. Here we only show the four plasmonic modes which exhibit mode bifurcation at low input power, although other modes may also be supported by the graphene coated nanowire.

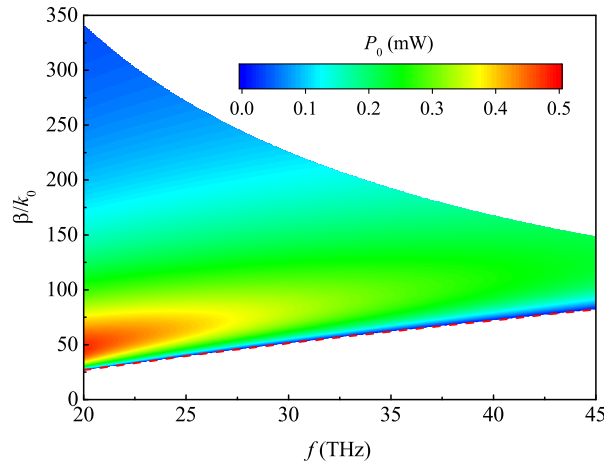


FIG. 4: Dispersion relation of the fundamental plasmonic mode with the dependence of the input power. For comparison, the red dashed curve shows the dispersion relation of the corresponding linear plasmonic mode. The parameters are $\mu_c = 0.3$ eV, $a = 100$ nm, $\varepsilon_1 = 3$, and $m = 0$.

Comparing the mode bifurcation curves with different orders, the limitation values for different plasmonic modes are not equal. At certain input powers, only the higher order modes exist, and the lower order modes vanish. This intriguing phenomenon is due to the self-action effect of graphene. Since the surface conductivity of graphene is dependent on the electric field that is parallel on the graphene surface, different field intensities are required to support the plasmonic modes with different orders, even at the same input power. Thus the nonlinear plasmonic waveguide may only support the higher order modes at certain input powers.

For the linear plasmonic modes in Refs. [32, 33], the dispersion relation between the propagation constant β and frequency f is a single curve, which is independent of the input power P_0 . However, for the nonlinear plasmonic modes, the dispersion curves at different input powers form an energy band, as shown in Fig. 4. For simplicity, we only consider the fundamental mode with $m = 0$, where the incident frequency is tuned between $f = 20$ THz and $f = 45$ THz to ensure the validity of the approximation conditions of $\hbar\omega \leq \mu_c$ and $\omega\tau \gg 1$.

Due to the nonlinearity of graphene, the plasmonic modes can only exist within a certain power range. For lower frequencies, the propagation constant can be tuned effectively by the input power. Whereas, the allowed band becomes narrow at high frequencies with the decrease of the limitation value of the input power. Actually, as the input power tends to zero, the dispersion relation reduces to that of the linear plasmonic mode, as shown by the red dashed curve in Fig. 4. Note the dispersion relation of the plasmonic mode would be different, if the realistic loss of graphene is considered. The further research is beyond the scope of this paper and we will show the relevant results elsewhere.

In conclusion, considering the vector nature of plasmonic modes in two dimensional waveguides, we derive the tensor form of nonlinear surface conductivity of graphene. The plasmonic modes with different orders are solved analytically in graphene-coated dielectric nanowire, where the propagation constant of each mode can be tuned effectively by the input power at the order of a few tenths of mW. The lower and upper mode bifurcation branches are connected at the limitation value of the input power. Moreover, due to the nonlinearity of graphene, the dispersion curves of plasmonic modes at different input powers form an energy band. Our work will provide important help to the research of other graphene-based nonlinear waveguides, especially plasmonic waveguide arrays and plasmonic lattices.

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- [1] W. L. Barnes, A. Dereux, and T. W. Ebbesen, *Nature* **424**, 824 (2003).
 - [2] M. Kauranen and A. V. Zayats, *Nat. Photon.* **6**, 737 (2012).
 - [3] V. M. Agranovich, V. S. Babichenko, and V. Ya. Chernyak, *Sov. Phys. JETP Lett.* **32**, 512 (1980).
 - [4] M. Y. Yu, *Phys. Rev. A* **28**, 1855 (1983).
 - [5] A. R. Davoyan, I. V. Shadrivov, and Y. S. Kivshar, *Opt. Exp.* **17**, 21732 (2009).
 - [6] A. Marini and D. V. Skryabin, *Phys. Rev. A* **81**, 033850 (2010).

- [7] D. Sarid, R. T. Deck, and J. J. Fasanot, J. Opt. Soc. Am. **72**, 1345 (1982).
- [8] G. I. Stegeman and C. T. Seaton, Opt. Lett. **9**, 235 (1984).
- [9] J. Ariyasu, C. T. Seaton, G. I. Stegeman, A. A. Maradudin, and A. F. Wallis, J. Appl. Phys. **58**, 2460 (1985).
- [10] E. Feigenbaum and M. Orenstein, Opt. Lett. **32**, 674 (2007).
- [11] A. Marini, D. V. Skryabin, and B. Malomed, Opt. Exp. **19**, 6616 (2008).
- [12] A. R. Davoyan, I. V. Shadrivov, and Y. S. Kivshar, Opt. Exp. **16**, 21209 (2008).
- [13] A. Marini, S. Roy, Ajit Kumar, and F. Biancalana, Phys. Rev. A. **91**, 043815 (2015).
- [14] Y. Liu, G. Bartal, D. A. Genov, and X. Zhang, Phys. Rev. Lett. **99**, 153901 (2007).
- [15] F. Ye, D. Mihalache, B. Hu, and N. C. Panou, Phys. Rev. Lett. **104**, 106802 (2010).
- [16] J. Yan, L. Li, and J. Xiao, Opt. Exp. **20**, 1945 (2012).
- [17] L. Wang, W. Cai, X. Zhang, and J. Xu, Opt. Lett. **37**, 2730 (2012).
- [18] Y. V. Bludov, D. A. Smirnova, Y. S. Kivshar, N. M. R. Peres, and M. I. Vasilevskiy, Phys. Rev. B **89**, 035406 (2014).
- [19] S. A. Mikhailov, Europhys. Lett. **79**, 27002 (2007).
- [20] S. A. Mikhailov and K. Ziegler, J. Phys. : Condens. Mat. **20**, 384204 (2008).
- [21] E. Hendry, P. J. Hale, J. Moger, A. K. Savchenko and S. A. Mikhailov, Phys. Rev. Lett. **105**, 097401 (2010)
- [22] K. L. Ishikawa, Phys. Rev. B **82**, 201402 (2010).
- [23] M. M. Glazov and S. D. Ganichev, Phys. Rep. **535**, 101 (2014).
- [24] M. L. Nesterov, J. B. Abad, A. Yu. Nikitin, F. J. G. Vidal, and L. M. Moreno, Laser Photon. Rev. **7**, L7 (2013).
- [25] H. Dong, C. Conti, A. Marini, and F. Biancalana, J. Phys. B **46**, 155401 (2013).
- [26] D. A. Smirnova, I. V. Shadrivov, A. I. Smirnov, and Y. S. Kivshar, Laser Photon. Rev. **8**, 291 (2014).
- [27] Y. V. Bludov, D. A. Smirnova, Y. S. Kivshar, N. M. R. Peres, and M. I. Vasilevskiy, Phys. Rev. B **91**, 045424 (2015).
- [28] N. M. R. Peres, Yu. V. Bludov, Jaime E. Santos, Antti-Pekka Jauho, and M. I. Vasilevskiy, Phys. Rev. B **90**, 125425 (2014).
- [29] L. Ju, B. Geng, J. Horng, C. Girit, M. Martin, Z. Hao, H. A. Bechtel, X. Liang, A. Zettl, Y. R. Shen, and F. Wang, Nat. Nanotech. **6**, 630 (2011).
- [30] K. I. Bolotin, K. J. Sikes, Z. Jiang, M. Klima, G. Fudenberg, J. Hone, P. Kim, H. L. Stormer, Solid State Commun. **146**, 351 (2008).
- [31] R. W. Boyd, *Nonlinear Optics*, 3rd ed. (Academic, San Diego, 2008).
- [32] Y. Gao, G. Ren, B. Zhu, H. Liu, Y. Lian, and S. Jian, Opt. Exp. **22**, 24322 (2014).
- [33] Y. Gao, G. Ren, B. Zhu, J. Wang, and S. Jian, Opt. Lett. **39**, 5909 (2014).